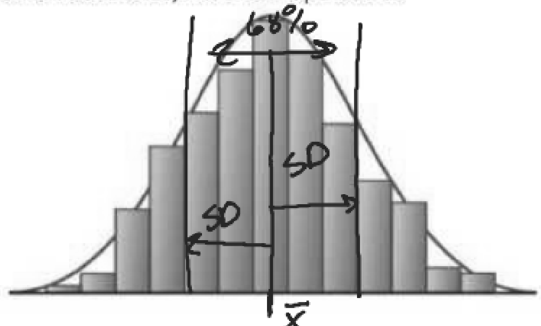
What you will learn about:
Characteristics of a Normal Distribution

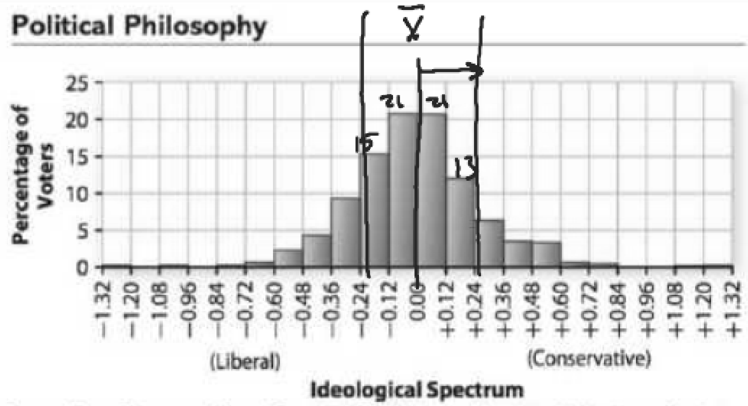
Many naturally occurring measurements, such as human heights or the lengths or weights of supposedly identical objects produced by machines, are **approximately normally distributed**. There histograms are "bell-shaped," with the data clustered symmetrically about the mean and tapering off on both ends, like the shape below.



When measurements can be modeled by a distribution that is approximately normal in shape, the mean and standard deviation often are used to summarize the distribution's center and variability.

1. In the past you estimated the mean of a distribution by finding the balance point of a histogram. You estimated the standard deviation of a normal distribution by finding the distance to the right and to the left of the mean that encloses the middle 68% (about two-thirds) of these values. Use this knowledge to help analyze the following situations.

   a. The histogram below shows the political points of view of a sample of 1,271 voters in the United States. The voters were asked a series of questions to determine their political philosophy and then were rated on a scale from liberal to conservative. Estimate the mean and standard deviation of this distribution.

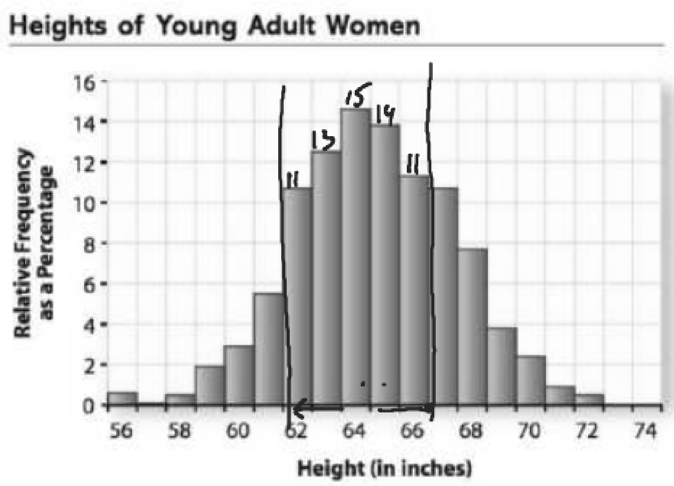


Source: Romer, Thomas, and Howard Rosenthal. 1984. Voting models and empirical evidence. *American Scientist*, 72: 465–473.

Mean: 0.00

Standard Deviation: .24

b. The relative frequency histogram below shows the heights (rounded to the nearest inch) of a large sample of young women in the United States. Estimate the mean and standard deviation of this distribution.



28
25
53
11
64

Mean: 64 in

Standard Deviation: 2 in

4.9941

4.939 – 5.0492
67

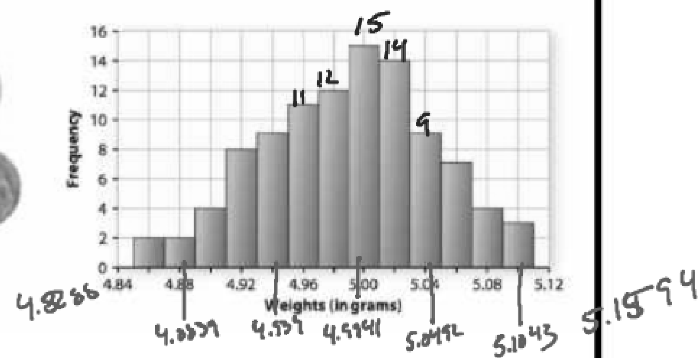
4.8839 – 5.1043
95

4.8288 – 5.1594
100

2. The data and the accompanying histogram give the weight, to the nearest hundredth of a gram, of a sample of 100 new nickels.

Nickel Weights (in grams)

4.87	4.92	4.95	4.97	4.98	5.00	5.01	5.03	5.04	5.07
4.87	4.92	4.95	4.97	4.98	5.00	5.01	5.03	5.04	5.07
4.88	4.93	4.95	4.97	4.99	5.00	5.01	5.03	5.04	5.07
4.89	4.93	4.95	4.97	4.99	5.00	5.02	5.03	5.05	5.08
4.90	4.93	4.95	4.97	4.99	5.00	5.02	5.03	5.05	5.08
4.90	4.93	4.96	4.97	4.99	5.01	5.02	5.03	5.05	5.09
4.91	4.94	4.96	4.98	4.99	5.01	5.02	5.03	5.06	5.09
4.91	4.94	4.96	4.98	4.99	5.01	5.02	5.04	5.06	5.10
4.92	4.94	4.96	4.98	5.00	5.01	5.02	5.04	5.06	5.11
4.92	4.94	4.96	4.98	5.00	5.01	5.02	5.04	5.06	5.11



a. The mean weight of this sample is 4.9941 grams. Find the median weight from the table above. How does it compare to the mean weight? med = 5.00g

b. Which of the following is the standard deviation?
.0253 grams .0551 grams .253 grams 1 gram

c. Mark points along the horizontal axis that correspond to the mean and one standard deviation above the mean, one standard devotion below the mean, two standard devotions above the mean, two standard deviations below the mean, three standard deviations above the mean, and three standard deviations below the mean.

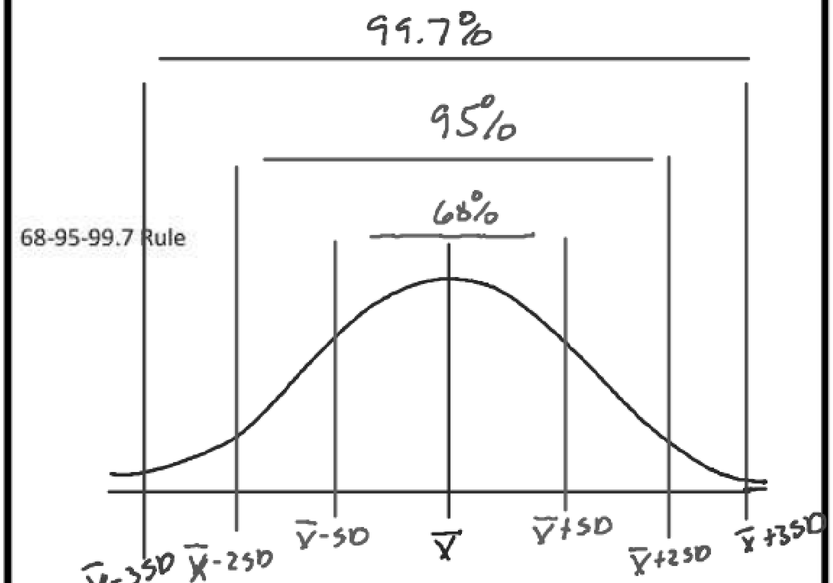
d. What percentage of the weights in the table above are within one standard deviation of the mean? Within two standard deviations? Within three standard deviations?

1 SD → $67/100 = 67\%$

2 SD → $95/100 = 95\%$

3 SD → $100/100 = 100\%$

e. Suppose you weigh a randomly chosen nickel from this collection. Find the probability that its weight would be within two standard deviations of the mean. 95%



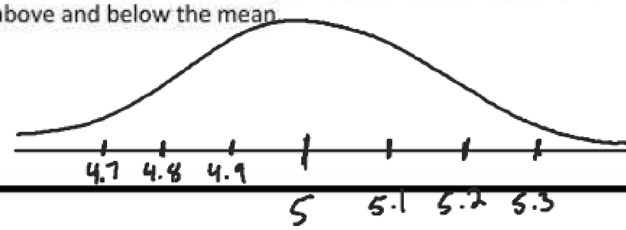
μ - Mu (mean)

σ - Sigma (SD)

(μ, σ)

3. Suppose that the distribution of the weights of newly minted coins is a normal distribution with a μ of 5 grams and standard deviation σ of 0.10 grams. (5, .10)

a. Draw a sketch of this distribution. Then label the point on the horizontal axis that corresponds to the mean, one standard deviation above and below the mean, two standard deviations above and below the mean, and three standard deviations above and below the mean.



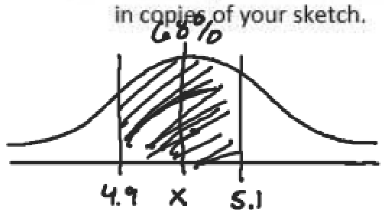
b. Between what two values do the middle 68% of the weights of coins lie? The middle 95% of the weights? The middle 99.7% of the weights?

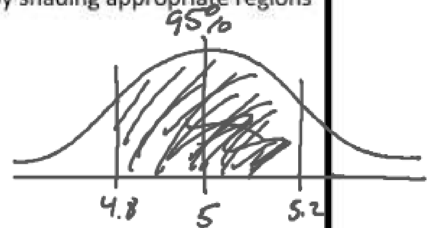
68% → 4.9 – 5.1

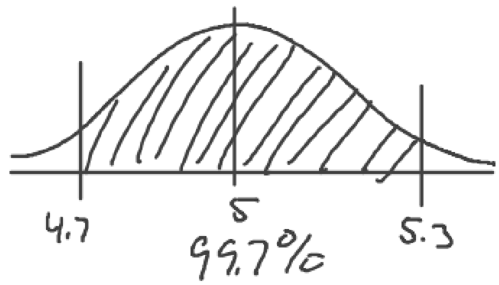
95% → 4.8 – 5.2

99.7 → 4.7 – 5.3

c. Illustrate your answers in Part b by shading appropriate regions in copies of your sketch.



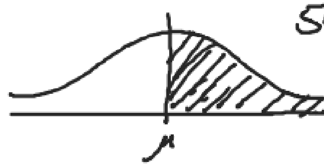




4. Answers the following questions about normal distributions. Draw sketches illustrating your answers.

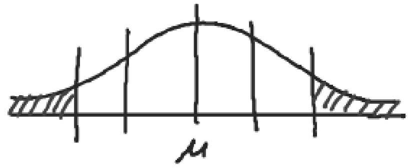
a. What percentage of the value in a normal distribution line above the mean?

50%

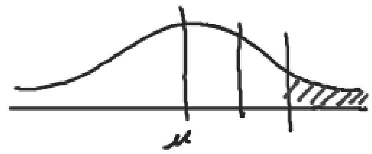


b. What percentage of the values in a normal distribution lie more than two standard deviations from the mean?

5%



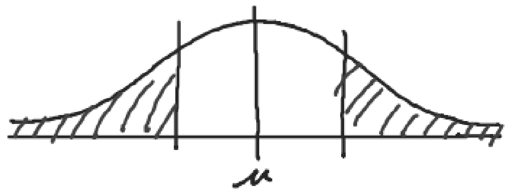
c. What percentage of the values in a normal distribution lie more than two standard deviations above the mean?



2.5%

d. What percentage of the values in a normal distribution lie more than one standard deviation from the mean?

32%



5. The weight of babies of a given age and gender are approximately normally distributed. This fact allows a doctor or nurse to use a baby's weight to find the weight percentile to which the child belongs. The table below gives information about the weights of six-month-old and twelve-month-old baby boys.

Weights of Baby Boys

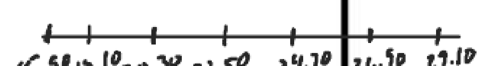
	Weight at Six Months (in pounds)	Weight at Twelve Months (in pounds)
Mean μ	17.25	22.50
Standard Deviation σ	2.0	2.2

Source: Tannenbaum, Peter, and Robert Arnold. *Excursions in Modern Mathematics*. Englewood Cliffs, New Jersey: Prentice Hall. 1992.

Six mo

a. On a separate axis, draw sketches that represent the distribution of weights for six-month-old boys and the distribution of weighs for twelve-month-old boys. How do the distributions differ?

12 mo



b. About what percentage of six-month-old weigh between 15.25 pounds and 19.25 pounds?